DTP/96/23

A Local Model of Explicit Wavefunction Collapse

Chris Dove[1]
and
Euan J. Squires[2]

Department of Mathematical Sciences
University of Durham
Durham City, DH1 3LE, England.

May 1996

Abstract

A model of spontaneous wavefunction collapse, which is explicitly local and Lorentz-invariant, is defined. Some of the predictions of the model for specific experimental situations are derived. It is shown that, although incompatible collapses, e.g. on opposite sides of an EPR-type of experiment, can occur, they will not persist in time and that eventually only compatible results will be obtained. The probabilities of particular results, however, will in general not agree with the predictions of quantum theory. We argue that it is unlikely that the deviations would have been seen in any experiment yet performed.

[1]E-mail:C.J.Dove@durham.ac.uk
[2]E-mail:E.J.Squires@durham.ac.uk

1 Introduction

In 1986 Ghirardi, Rimini and Weber [1] showed that it was possible to construct a realistic model describing explicit wavefunction collapse in such a way that, in many situations, the correct predictions of quantum theory were maintained but real experiments actually had results. Their work has since been developed in a number of ways [2] and it is generally agreed that it provides a satisfactory resolution of the measurement problem of quantum theory, at least in the non-relativistic domain. As originally presented, however, the model was clearly non-local and not Lorentz invariant. Recently, attempts have been made to develop versions of the collapse models which, whilst retaining the non-locality, are nevertheless Lorentz invariant[3, 4, 5]. Perhaps the best one can say of these models is that they are partially successful. They certainly raise several interesting issues.

In this work we shall take a different approach and endeavour to construct a *local*, and Lorentz invariant version of the collapse model. We know of course that this cannot agree in all respects with the predictions of orthodox quantum theory, and it is one object of this work to see where the disagreement lies and whether it is detectable. Note that even the original GRW model does not completely agree with quantum theory, and this requires severe constraints to be placed on the parameters [6, 7, 8]. We are concerned here with a different type of departure from quantum theory, which is caused by our insistence on the theory being local.

2 A local model of collapse

In the original GRW model, it was proposed that 'hits' occurred in a random fashion, at certain space-time points. The effect of a given hit spread throughout all space instantaneously. Thus, if we have a single particle wavefunction $\psi(\mathbf{x}, t)$, a hit at the point $\mathbf{x}_1$, would cause this to change according to:

$$\psi \rightarrow \psi' = N \exp\left(-\frac{\beta}{2}(\mathbf{x} - \mathbf{x}_1)^2\right)\psi. \tag{1}$$

In order to make this into something that is both local and Lorentz invariant, we propose instead that a hit at the space-time point $\mathrm{X}_1 \equiv (\mathbf{x}_1, t_1)$ only has an effect inside the forward light-cone from that point. To ensure Lorentz invariance of the hitting function, we must replace the 3-dimensional distance in eq. (1) by a four-dimensional distance. We cannot use the distance from the hitting point to the point on the light-cone since this is identically zero. Instead, we propose the perpendicular distance from the point on the light-cone to a four-momentum vector P_μ originating from X, where perpendicular is meant in the sense of a Minkowski metric. With $\psi(\mathbf{x}, t) \equiv \psi(\mathrm{X})$, we

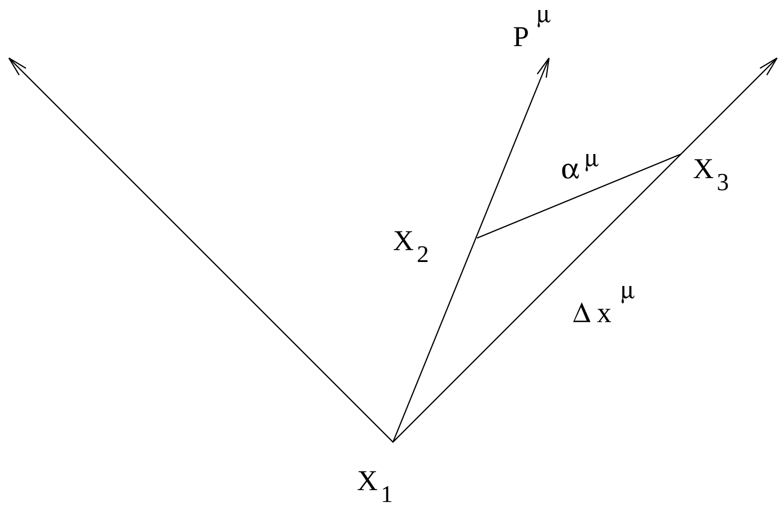


Figure 1: Constructing a Lorentz invariant distance

define this momentum vector by:

$$\mathrm{P}^{(1)}_\mu = \Re\left(\frac{p^{op}_\mu \psi}{\psi}\right)_{\mathrm{X}=\mathrm{X}_1}. \tag{2}$$

If the particle is in an eigenstate of momentum, then this formula will just give the four-momentum of the particle. More generally it is the 4-vector form of the particle momentum used in the Bohm hidden-variable model.

If we denote the vector from the light-cone to P^μ by α^μ (see Fig. 1) then the condition that it is perpendicular to P^μ is

$$\mathrm{P}_\mu \alpha^\mu = 0. \tag{3}$$

The path from X_1 to X_3 can be traversed in two ways, giving another condition

$$k\mathrm{P}^\mu + \alpha^\mu = \Delta \mathrm{x}^\mu, \tag{4}$$

for some k. These two equations enable us to find the value of $\alpha_\mu \alpha^\mu$. From eq. (2), and using eq. (3), we have

$$k\mathrm{P}_\mu \mathrm{P}^\mu = \mathrm{P}_\mu \Delta \mathrm{x}^\mu, \tag{5}$$

and

$$\alpha_\mu \alpha^\mu = \alpha_\mu \Delta \mathrm{x}^\mu. \tag{6}$$

Also, since $\Delta \mathrm{x}^\mu$ is a null vector, eq. (4) gives

$$k\Delta \mathrm{x}_\mu \mathrm{P}^\mu + \Delta \mathrm{x}_\mu \alpha^\mu = 0. \tag{7}$$

We can rearrange these three equations to eliminate k and, putting $\mathrm{P}_\mu \mathrm{P}^\mu = m^2c^2$, we have

$$\alpha^\mu = \Delta \mathrm{x}^\mu - \frac{1}{m^2c^2}\mathrm{P}^\mu \left(\mathrm{P}_\nu \Delta \mathrm{x}^\nu\right), \tag{8}$$

and

$$\alpha_\mu \alpha^\mu = -\frac{1}{m^2c^2}\left(\mathrm{P}_\mu \Delta \mathrm{x}^\mu\right)^2. \tag{9}$$

This reduces to $\alpha_\mu \alpha^\mu = -(\Delta \mathbf{x})^2$ in the rest frame of the particle, $\mathrm{P}_\mu = (mc, \mathbf{0})$.

We therefore postulate that the collapse takes effect along the forward light cone from X_1, according to

$$\psi_{H_1}(\mathrm{X}) = \exp\left(-\frac{\beta}{2m^2c^2}\left(\mathrm{P}^{(1)}_\mu(\mathrm{X}^\mu - \mathrm{X}^\mu_1)\right)^2\right)\psi(\mathrm{X}). \tag{10}$$

This is our local analogue of eq. (1).

In what follows, we shall simplify the discussion by constraining the particle to a single spatial dimension (z). Ideally, we should take the wavefunction to be a solution of the Dirac equation. However, we wish not to be concerned with any Dirac bispinor, as the collapse process does not act on the space of spins. For a free particle, we can instead take the wavefunction to be a solution of the Klein-Gordon equation. We shall work with a single momentum for which the initial wavefunction is

$$\psi_0(t, z) = N \exp\left(-iEt + ipz\right), \tag{11}$$

where N is some normalization factor.

Given that the forward light-cone is the boundary under consideration, it is sensible to use light-cone coordinates, $x_+ = ct + z$, $x_- = ct - z$. The Klein-Gordon equation in this coordinate system reads

$$\frac{\partial^2}{\partial x_+ \partial x_-}\psi = -\frac{1}{4}\left(\frac{mc}{\hbar}\right)^2 \psi. \tag{12}$$

Then, if we choose the origin to be at the point of collapse, the boundary conditions in a general frame of reference are

$$\begin{aligned}
\psi(x_+, 0) &= N \exp\left(-\frac{i}{2}(E-p)x_+\right)\exp\left(-\frac{\beta}{8m^2}(E-p)^2 x_+^2\right) && (13)\\
\psi(0, x_-) &= N \exp\left(-\frac{i}{2}(E+p)x_-\right)\exp\left(-\frac{\beta}{8m^2}(E+p)^2 x_-^2\right). && (14)
\end{aligned}$$

The solution of the Klein-Gordon equation inside the forward light-cone from the point of collapse is uniquely defined by these boundary conditions. In order to be able to write this down in a simple form, we shall ignore the quantum evolution, i.e.

assume $\frac{\hbar}{mc}$ is very small. For simplicity, we work in the rest frame, in which $p = 0$. Then we can write

$$\psi(x_+, x_-) = N \exp\left(-\frac{im}{2}(x_+ + x_-)\right) w(x_+, x_-). \tag{15}$$

Substituting this expression into the Klein-Gordon equation, we have

$$\frac{\partial w}{\partial x_+} + \frac{\partial w}{\partial x_-} = -\frac{2i\hbar}{mc}\frac{\partial^2 w}{\partial x_+ \partial x_-}, \tag{16}$$

where we have included all constants which had been previously set to unity. The right-hand-side is responsible for the quantum evolution. It can be treated as a perturbation [9]. Here we shall ignore it and just use the zeroth order solution which is $w(x_+, x_-) = h(x_+ - x_-)$. Substituting in the boundary conditions leads us to a solution:

$$\psi(t, z) = N \exp(-imt) \exp\left(-\frac{\beta}{2} z^2\right), \tag{17}$$

within the forward light-cone of X. Outside of this region, the original free-particle solution holds.

If we take the initial wavefunction to be a gaussian with a large spread, $\psi_0 \sim \exp\left(-\frac{z^2}{a^2}\right)$, with $a \gg \frac{\hbar}{mc}$, then the momentum states contributing will have $p < \frac{\hbar}{a} \ll mc$. We should note that using the collapse radius for a here gives $p < 2 \times 10^{-9} mc$, and we are justified in taking this to have a single momentum component.

To summarise this section, the effect of a single collapse on a single particle is the same as in the non-relativistic case, except that the effect is only felt within the forward light-cone of the point of origin of the collapse, X_1.

3 Single particle affected by two collapses

A major difference between our local collapse model and that of GRW is that two independent collapses can occur at space-like separations so that neither collapse 'knows about' the other. There is no problem with consistency until we arrive at the intersection of the light-cones arising from the two collapse centers. In the region formed by the forward light-cone originating from the point of intersection both collapses will be felt, and we need to define precisely how this happens.

We shall take the two collapses to occur at the points $\mathrm{X}_1 \equiv (z_1, t_1) \equiv (x_{1+}, x_{1-})$ and $\mathrm{X}_2 \equiv (z_2, t_2) \equiv (x_{2+}, x_{2-})$, see Fig. 2. Before the point of intersection of the two light-cones, the wavefunction in the regions w_1 and w_2 will be calculated as in the previous section. After we reach the intersection point, $\mathrm{X}_3 \equiv (x_{2+}, x_{1-})$, we shall solve the differential equation again with new boundary conditions along this third light-cone.

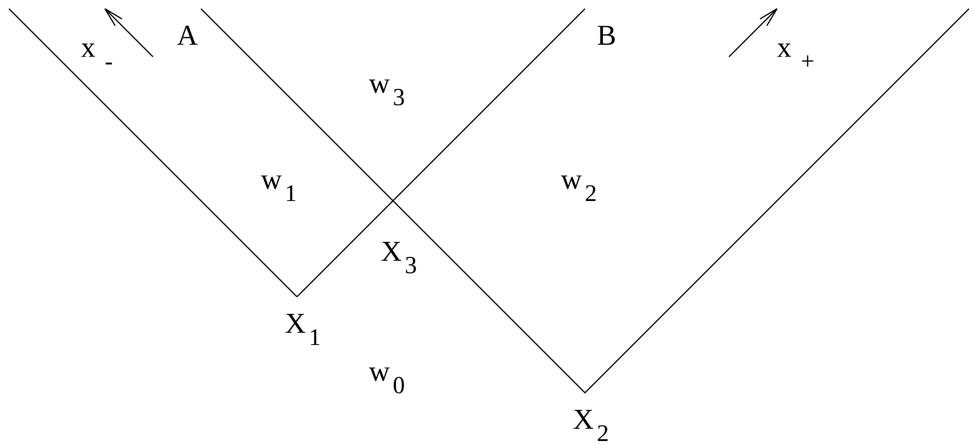


Figure 2: Wavefunction affected by two collapses

The boundary conditions are formed, as indeed they were before, by taking the wavefunction outside of the light-cone, and multiplying by the collapse factor which arises from the collapse along that particular light-cone. Thus, for the boundary condition along X_3A we take the wavefunction in the region w_1, which is $\psi_{w_1}(z,t)$, and multiply by the collapse factor arising from the collapse at X_2 along X_2A. Hence

$$\psi_{\mathrm{X}_1\mathrm{X}_2}(z,t) = \exp\left(-\frac{\beta}{2}\left(\alpha_{\mathrm{X}_2}\right)^2\right)\psi_{w_1}(z,t), \tag{18}$$

and simliarly along X_3B

$$\psi_{\mathrm{X}_2\mathrm{X}_1}(z,t) = \exp\left(-\frac{\beta}{2}\left(\alpha_{\mathrm{X}_1}\right)^2\right)\psi_{w_2}(z,t), \tag{19}$$

where α_{X_1} and α_{X_2} are the perpendicular four-distances from the momentum vectors arising from the collapses at X_1 and X_2 respectively.

In general, for an arbitrary initial wavefunctions the momentum vectors arising from each point will be different. Even ignoring the quantum evolution, this could lead to solutions of the differential equation which are quite complicated. For simplicity, we shall deal with the case when the momenta arising from each collapse center are equal. In this situation, we can again work in the frame where $\mathbf{p}=0$. This means that the boundary conditions are (having extracted the plane-wave term and normalization as before):

$$w(x_{2+},x_-) = \exp\left(-\frac{\beta}{2}(z-z_1)^2\right)\exp\left(-\frac{\beta}{8}(x_- - x_{2-})^2\right), \tag{20}$$

along $X_3 A$, and

$$w(x_+, x_{1-}) = \exp\left(-\frac{\beta}{2}(z - z_2)^2\right) \exp\left(-\frac{\beta}{8}(x_+ - x_{1+})^2\right), \tag{21}$$

along $X_3 B$.

We are only interested in the zeroth order solution to the differential equation, eq. (16), so of course we have $w(x_+, x_-) = h(x_+ - x_-)$, as before. Substituting the boundary conditions, we find that

$$h(x_+ - x_{1-}) = \exp\left(-\frac{\beta}{8}\left[(x_+ - x_{1-} - x_{2+} + x_{2-})^2 + (x_+ - x_{1+})^2\right]\right), \tag{22}$$

which may be rewritten as

$$h(\lambda) = \exp\left(-\frac{\beta}{8}\left[(\lambda - x_{2+} + x_{2-})^2 + (\lambda - x_{1+} + x_{1-})^2\right]\right), \tag{23}$$

leading to a wavefunction

$$\psi(t, z) = N(0, m) \exp\left(-i\frac{mc^2}{\hbar}t\right) \exp\left(-\frac{\beta}{2}(z - z_1)^2\right) \exp\left(-\frac{\beta}{2}(z - z_2)^2\right). \tag{24}$$

Here we find that, in this case, the two collapses are equivalent to a single collapse at the point $\frac{1}{2}(X_1 + X_2)$, but with twice the 'strength'. This is certainly what would be expected in the non-relativistic limit if we were to have two collapses, although this solution only holds in the forward light-cone of the intersection point X_3. It should be noted that in the non-relativistic situation, the wavefunction at the point of the second collapse would have already been reduced by the first, so the probability of the second collapse occurring would be very small.

We now briefly consider the question of the order of the two collapses. Take the situation shown in fig. 3, where the collapse at X_1 happens later than the collapse at X_2, in the frame in which $p = 0$. The relative size of the two peaks depends on the distances from X_1 to A_2 and X_2 to A_1. As can be seen from the diagram, these distances are of course equal, so the exponentials by which we multiply the two wavefunctions will be equal, at the two peaks, and thus as the wavefunction is not time-dependent, the two peaks will have the same size. The order of the collapses is immaterial when we have a single momentum component.

3.1 Superposition of Two Wavepackets

We now want to examine a typical measurement situation, where the initial wavefunction is a sum of two well-separated peaks, e.g.

$$\psi_0(z) = N\left[\exp\left(-\alpha(z - z_1)^2\right) + \exp\left(-\alpha(z - z_2)^2\right)\right], \tag{25}$$

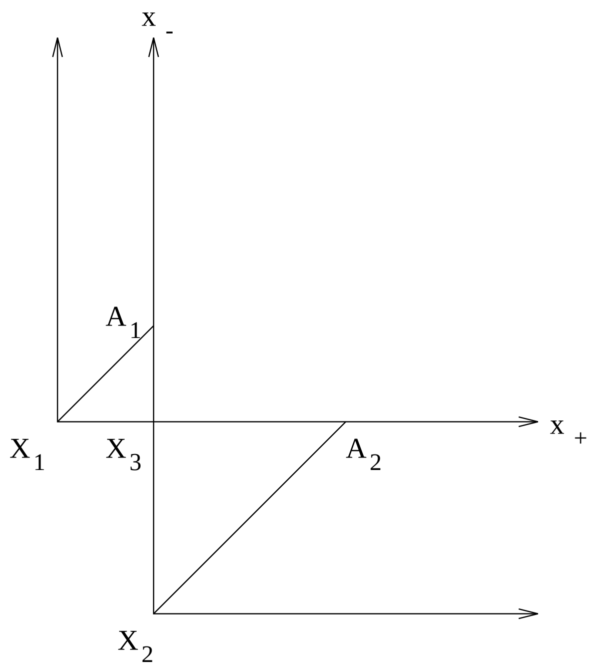


Figure 3: The effect of the relative times of the collapses

with $|z_1 - z_2| \gg \frac{1}{\sqrt{\alpha}}$. So any collapse which occurs will, with probability essentially one, be centered around one of the two peaks of the wavefunction. An important property of the GRW-type models which we retain is that the probability of a collapse occurring at a point $\mathbf{x}$ is proportional to $|\psi(\mathbf{x})|^2$; with a single collapse, the wavefunction will be reduced to a single peak in a time $t \approx \frac{|z_1 - z_2|}{c}$.

However, as before, the relativistic model allows the possibility of there being two collapse events, one centered on each peak, providing that each collapse event is outside of the forward light-cone of the other. On a constant time slice, there may persist peaks in each region, but we are predominately concerned with the shape of the wavefunction at later times, i.e. after the intersection of the two forward light-cones.

We assume that the momentum vectors defined at the two collapse points are (to a sufficiently good approximation) the same, so that we can again work in the frame for which $\mathbf{p} = 0$. Then, with the same approximations as before, the final state wavefunction will just be the initial state wavefunction multiplied by the collapse functions arising from the spatially separated collapses. This final state wavefunction can be written as

$$\psi_f = N \left[\exp\left(-(\alpha + \beta)(z - z_1')^2 \right) + \exp\left(-(\alpha + \beta)(z - z_2')^2 \right) \right], \tag{26}$$

and we again have the two peaks, only now their centers have been shifted, according to

$$z_1' = z_1 + \frac{\frac{1}{2}\beta}{\alpha+\beta}(z_2 - z_1) \tag{27}$$

$$z_2' = z_2 + \frac{\frac{1}{2}\beta}{\alpha+\beta}(z_1 - z_2). \tag{28}$$

Obviously if the peaks were very sharp in the initial wavefunction, then the shift will be quite small. However, it is certainly possible that the shift will be sufficient for the new peak to lie well into the tail of the initial peak, where it would have been extremely unlikely that the particle could be found. This will be the case if $\exp\left(-\frac{\alpha\beta^2}{4(\alpha+\beta)^2}(z_2 - z_1)^2\right) \ll 1$. For a pair of sharp initial peaks, and $\alpha \gg \beta$, this reads $|z_2 - z_1| \gg \frac{2\sqrt{\alpha}}{\beta}$.

In general we might expect the localization of the two peaks to be less than, but of the order of, the GRW collapse size, i.e. $\beta < \alpha$ but of the same order. This means that the peaks are shifted by something around $\frac{1}{4}$ of their separation.

4 The Born Probability Rule

In orthodox quantum theory, the probability that a measurement outcome will correspond to one of 2 peaks is proportional to the square integral of the weight of each peak. The same result holds in GRW, where it is a consequence of the probability rule for a hit occurring at a particular point [1]. Here we shall again guarantee this result, *for a single collapse*, by postulating that the probability of this collapse occurring at one of the peaks is proportional to the integral of the square of the wavefunction over the peak. Thus with an initial state (in the rest frame):

$$\psi_0(z) = N\left[a\exp\left(-\alpha(z - z_1)^2\right) + b\exp\left(-\alpha(z - z_2)^2\right)\right], \tag{29}$$

with $|a|^2 + |b|^2 = 1$, the collapse will occur near z_1 or z_2 in the ratio of $|a|^2$ to $|b|^2$.

However, we now have to consider carefully the possibility of more than one collapse occurring. This means of course that both peaks can change their magnitudes. We take account of this by allowing a and b in eq. (30) to be functions of time. Consistent with the requirement of a *local* model we postulate that the probability of a collapse at z_1 at time t_1 is proportional to $\frac{|a(t)|^2}{|a(t)|^2+|b(t_R)|^2}$, where $t_R = t_1 - \frac{|z_1 - z_2|}{c}$, the retarded time.

The probability of a particular peak persisting depends of course on the number of collapses that occur, and the time taken for the signal of a collapse to reach the other peak(T). Here we shall evaluate the probability of peak 1 dominating. There will be contributions to this probability from all possible numbers of collapses. We

shall assume that $\lambda T \ll 1$, and so make an expansion in this parameter. We shall calculate the first three terms in this series, i.e. work to order $(\lambda T)^2$.

The collapse processes which contribute to this order are illustrated in Fig. 4. The separation of the two peaks is $z_1 - z_2 \equiv L \equiv Tc$. In order to assess the probability of a collapse occuring on a particular peak, we look at the relative sizes of the peaks along the backward light-cone. In the figure, the solid vertical lines indicate where a collapse on the peak is possible, whereas the dotted lines indicate that a collapse is not possible. When a collapse is deemed possible, the probabilities for each side will be $|a|^2$ and $|b|^2$ respectively, if both can occur, or 1 and 0 if only one of these is possible. We need to calculate the probability contributions from each diagram separately.

Diagram (i) In this case a single collapse is successful. There are no collapses on the other peak before it has received the signal from the first collapse. The probability of this occurring is

$$P_i = |a|^2 \exp\left(-\lambda T|b|^2\right) = |a|^2 \left(1 - \lambda T|b|^2 + \frac{1}{2}(\lambda T)^2|b|^4 + O([\lambda T]^3)\right). \tag{30}$$

Diagram (ii) Here we have two specified collapses, C_1 and C_2, one centered on each peak, with that on the peak number 1 occurring first, and with no further collapses before the time indicated by the dashed line. The probability can be written as

$$\begin{aligned} P_{ii} &= |a|^2 \int_0^T \exp\left(-\lambda T|b|^2\right) \lambda dt|b|^2 \exp\left(-\lambda(T+t)|a|^2\right) \exp\left(-2\lambda T\right) \exp\left(-\lambda t|b|^2\right) P \\ &= |a|^2|b|^2(\lambda T)[1 - \frac{7}{2}(\lambda T)]\, P + O([\lambda T]^3), \end{aligned} \tag{31}$$

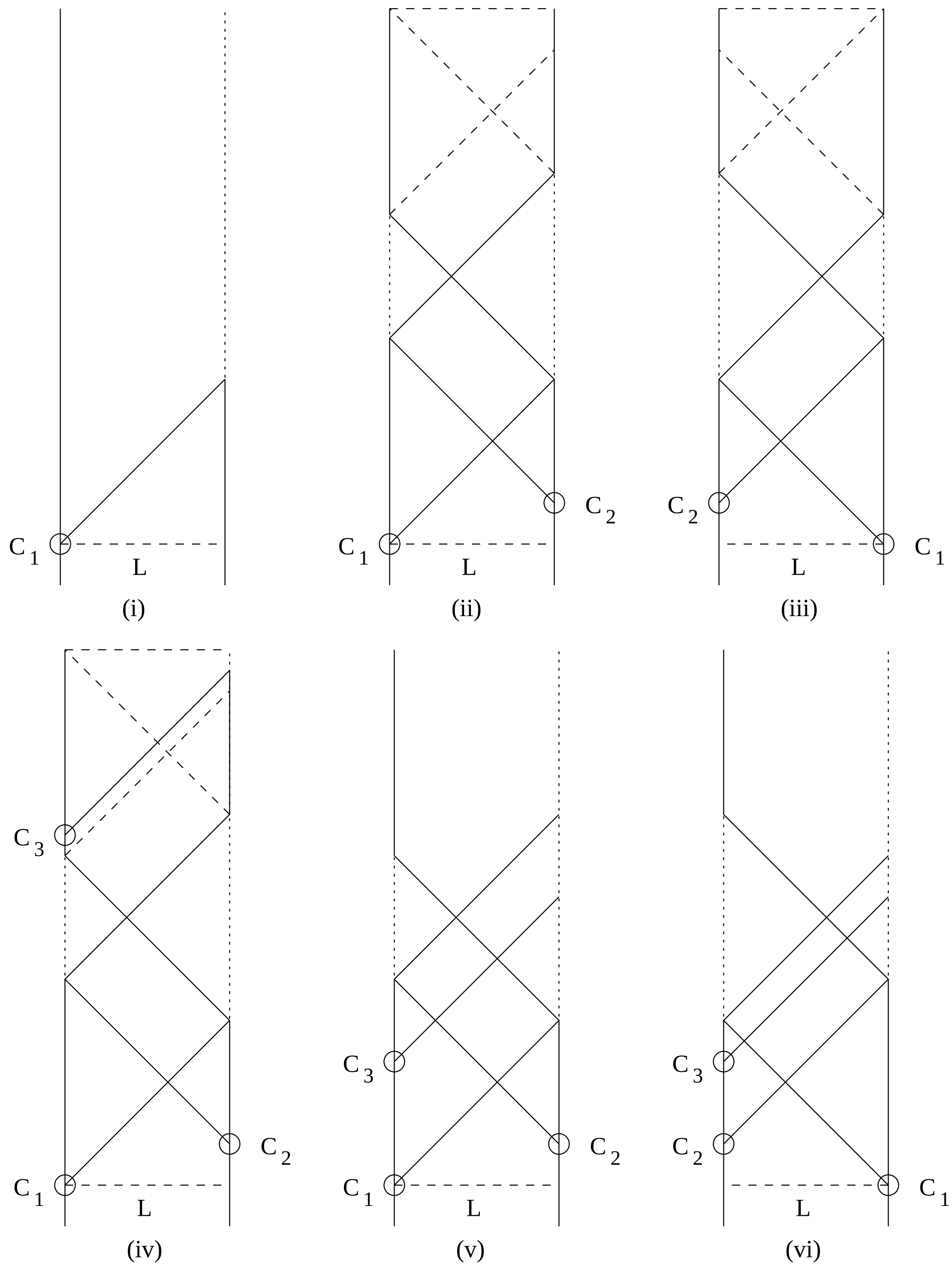
where P is the overall probability of peak 1 dominating.

Diagram (iii) This diagram is a mirror image of diagram (ii), and the probability contribution is in fact the same.

$$P_{iii} = P_{ii}. \tag{32}$$

Diagram (iv) The initial collapses are the same as in diagram (ii), only we have a further collapse occurring before the dashed line. (n.b. if there are no further collapses before this time, then we essentially have the situation with which we started.) This added collapse gives rise to another integral in the calculation of the probability

$$\begin{aligned} P_{iv} &= |a|^2 \int_0^T \exp\left(-\lambda T|b|^2\right) \lambda dt|b|^2 \exp\left(-\lambda(T+t)|a|^2\right) \\ &\qquad \times \int_0^{T+t} \exp\left(-2\lambda T\right) \lambda dt' \exp\left(-\lambda t'|b|^2\right) \\ &= \frac{3}{2}(\lambda T)^2|a|^2|b|^2 + O([\lambda T]^3). \end{aligned} \tag{33}$$


C 1
L
(i)
C 1
C 2
L
(ii)
C 2
C 1
L
(iii)
C 3
C 2
C 1
L
(iv)
C 3
C 2
C 1
L
(v)
C 3
C 2
C 1
L
(vi)

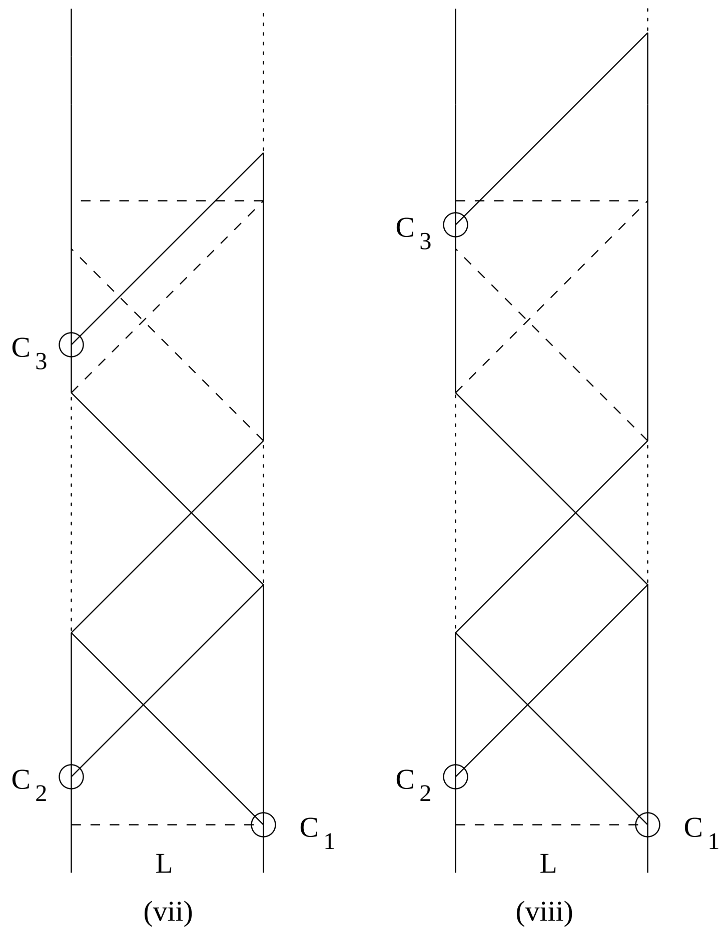


Figure 4: Collapse processes which contribute to second order in λT.

Diagram (v) This time we have two collapses centered on peak 1 before the collapse on the second peak has taken full effect, with the first collapse centered on peak 1.

$$\begin{aligned} P_v &= |a|^2 \int_0^T \exp\left(-\lambda T|b|^2\right) \lambda dt |b|^2 \lambda |a|^2 (T+t) \exp\left(-\lambda(T+t)|a|^2\right) \\ &= \frac{3}{2}(\lambda T)^2 |a|^4 |b|^2 + O([\lambda T]^3). \end{aligned} \tag{34}$$

Diagram (vi) This is similar to the previous diagram, with the solitary collapse on peak number 2 occurring first.

$$\begin{aligned} P_{vi} &= |b|^2 \int_0^T \exp\left(-\lambda T|a|^2\right) \lambda dt |a|^2 \exp\left(-\lambda(T+t)|b|^2\right) \int_t^T \lambda dt' |a|^2 \\ &= \frac{1}{2}(\lambda T)^2 |a|^4 |b|^2 + O([\lambda T]^3). \end{aligned} \tag{35}$$

Diagram (vii) This diagram is similar to diagram (iii) in the same way as diagram (iv) is related to diagram (ii).

$$\begin{aligned} P_{vii} &= |b|^2 \int_0^T \exp\left(-\lambda T|a|^2\right) \lambda dt |a|^2 \exp\left(-\lambda(T+t)|b|^2\right) \\ &\qquad\qquad \int_0^{T-t} \lambda dt' \exp\left(-2\lambda T\right) \exp\left(-\lambda t'|b|^2\right) \\ &= \frac{1}{2}|a|^2|b|^4(\lambda T)^2 + O([\lambda T]^3). \end{aligned} \tag{36}$$

Diagram (viii) This is almost the same as the last diagram, except that the time of the last collapse gives it a different probability of occurring.

$$\begin{aligned} P_{viii} &= |b|^2 \int_0^T \exp\left(-\lambda T|a|^2\right) \lambda dt |a|^2 \exp\left(-\lambda(T+t)|b|^2\right) \\ &\qquad \int_0^{t} \lambda dt' |a|^2 \exp\left(-2\lambda T\right) \exp\left(-\lambda t|a|^2\right) \exp\left(-\lambda(T-t+t')|b|^2\right) \\ &= \frac{1}{2}|a|^4|b|^2(\lambda T)^2 + O([\lambda T]^3). \end{aligned} \tag{37}$$

Adding all these calculated probabilities together and rearranging, leads to

$$P = |a|^2 + \lambda T|a|^2|b|^2(|a|^2-|b|^2) - \frac{1}{2}(\lambda T)^2|a|^2|b|^2(|a|^2-|b|^2)(5-4|a|^2|b|^2). \tag{38}$$

If the initial superposition is equally weighted, then as expected the probabilities for each peak to dominate are equal. However, if we start with an unequal superposition of the two gaussian peaks, then in this model the probability of obtaining the initially higher peak increases with the peak separation.

5 Two-particle correlated wavefunction

We will now deal with the case where we have two particles with a correlated wavefunction, for instance an EPR-type situation corresponding to the measurement of the spins of two fermions in a correlated state [3]. The initial wavefunction can be written in the form:

$$\begin{aligned} \psi(z_1,z_2) &= N\left[a\phi_1(z_1)\phi_2(z_2) + b\chi_1(z_1)\chi_2(z_2)\right] \\ &= N\Big[a \exp\left(-\alpha(z_1-z_{11})^2\right) \exp\left(-\alpha(z_2-z_{21})^2\right) \\ &\qquad + b\exp\left(-\alpha(z_1-z_{12})^2\right) \exp\left(-\alpha(z_2-z_{22})^2\right)\Big], \end{aligned} \tag{39}$$

[3]It should be noted that there is now an ambiguity in the definition of P_μ (see eq. (2)) since the right-hand-side depends upon the value of the position variable for the other particle. The simplest procedure would be to say that the collapse selects a random value for this, i.e. that it is actually associated with a point in configuration space. This issue does not concern us here since we are restricting our attention to situations where all '3-momenta' are, to a sufficiently good approximation, zero in some reference frame.

where z_{11}, z_{12} refer to the center of the peaks corresponding to particle 1, and similarly for particle 2. We assume that the two peaks for each particle do not overlap significantly, so that a collapse centered on one will kill the other peak, i.e. $\alpha(z_{11}-z_{12})^2 \gg 1$ and $\beta(z_{11}-z_{12})^2 \gg 1$. Also, for simplicity, we will consider only the case when this peak separation itself is negligible compared with the separation of the two particles, for instance, $|z_{11}-z_{21}| \gg |z_{11}-z_{12}|$. This of course corresponds to the actual experimental situations in tests of the Bell inequality.

Collapse processes centered on each particle will be taken to be independent, and they will have the same effect on the wavefunction as before. However, in the case where we have two 'incompatible' collapses, the situation will have changed in that as each collapse acts on a different part of the wavefunction, they will not 'interfere' at any point in space. At a time after signals from both collapses have reached the other, the wavefunction in the intermediate region will just be multiplied by the two independent collapse factors irrespective of the momentum states from which they were constructed, i.e. for collapses centered at z_{11} and z_{22},

$$\psi' = \psi \exp\left(-\frac{\beta}{2}(z_1 - z_{11})^2\right) \exp\left(-\frac{\beta}{2}(z_2 - z_{22})^2\right). \tag{40}$$

As before, we should again ask which part of the wavefunction will dominate. We can do a similar calculation to before, with the probability of a collapse occurring on a particular peak being either $|a|^2$, $|b|^2$, 1 or zero. However, as the peak separation of either particle is considered to be negligible compared to the separation between the two particles, we shall take the signal of a collapse to travel instantaneously to the other peak connected to that particle. For a single particle, the probability of another collapse in the actual time taken is very small.

It turns out that the probability for a particular peak to dominate is actually the same as when we only have one particle.

Fig. 5 illustrates the diagrams which contribute to second order in λT, ignoring the separation between the two peaks on one side. As before the solid lines indicate that a collapse is deemed possible, whereas a collapse cannot occur where the line is dotted. The points where a collapse occurs are circled. The peaks for particle 1 are those to the left, with the ψ peak the right one of these and the χ peak on the left. We calculate the probability of the ψ peaks dominating.

Diagram (i) The simplest case where we have no incompatible collapses.

$$\begin{aligned} P_{2i} &= |a|^2 \left[\exp(-\lambda T) + \int_0^T \lambda |a|^2 dt \exp(-\lambda t)\right] \\ &= |a|^2 \left(1 - \lambda T|b|^2 + (\lambda T)^2 |b|^2\right) + O([\lambda T]^3). \end{aligned} \tag{41}$$

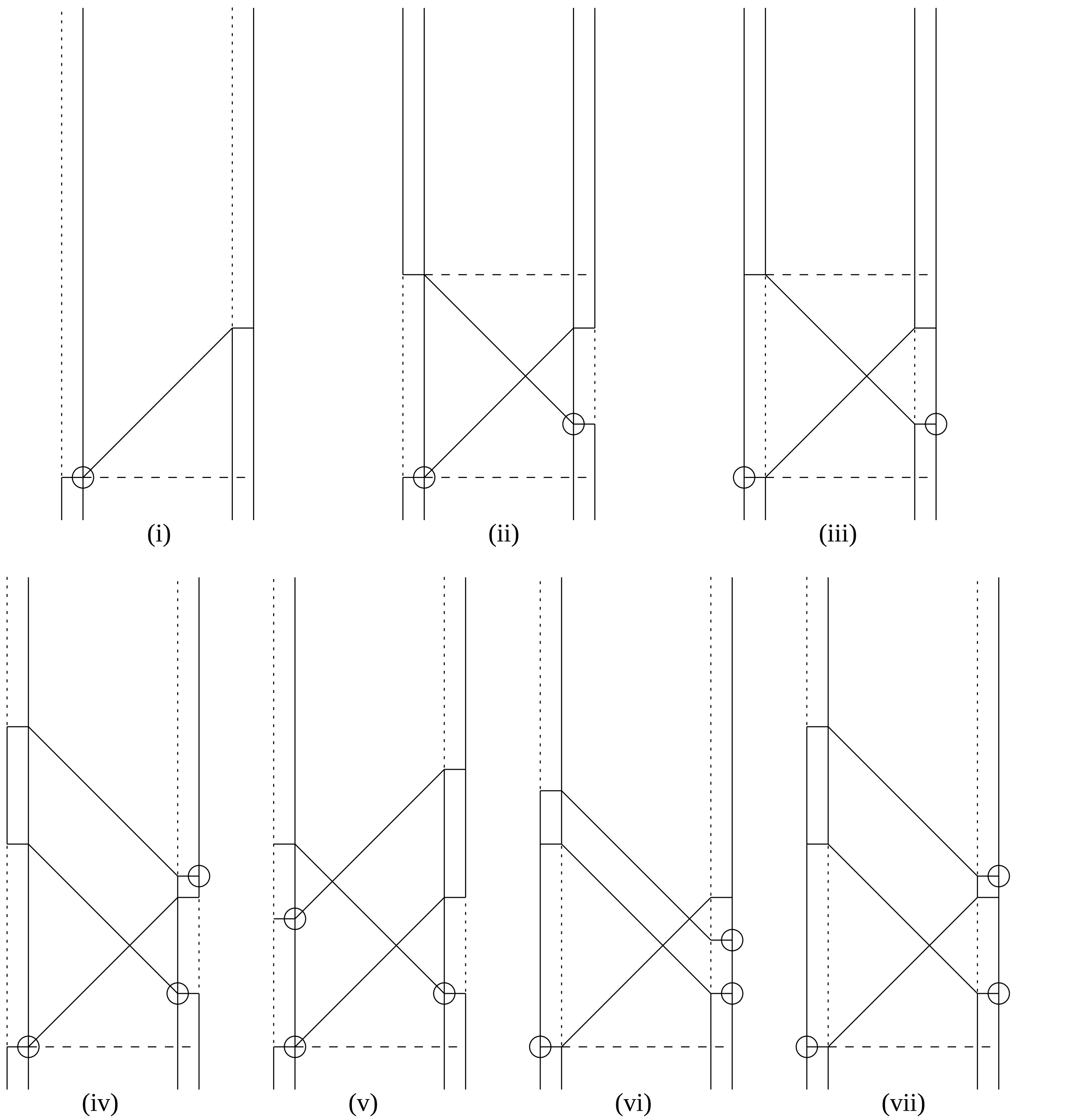


Figure 5: The collapse processes which contribute to second order in λT for two correlated particles.

Diagram (ii) As in the one-particle case, we have two specified collapses, one affecting each particle, but incompatible.

$$\begin{aligned} P_{2ii} &= |a|^2 \int_0^T \lambda |b|^2 dt \exp\left(-2\lambda(T+t)\right) P_2 \\ &= |a|^2 |b|^2 (\lambda T) \left(1 - 3(\lambda T)\right) P_2 + O([\lambda T]^3), \end{aligned} \tag{42}$$

where P_2 is the overall probability of the ψ peaks dominating for this two-particle wavefunction.

Diagram (iii) This is just the mirror image of the previous diagram, with identical contribution to the probability.

$$P_{2iii} = P_{2ii}. \tag{43}$$

Diagram (iv) As in diagram (ii) but with an additional collapse prior to both particles having knowledge of both previous collapses.

$$\begin{aligned} P_{2iv} &= |a|^2 \int_0^T \lambda |b|^2 dt \int_0^t \lambda |a|^2 dt' \exp\left(-\lambda(T+t')\right) \exp\left(-\lambda(T-t+t')\right) \\ &= \frac{1}{2} |a|^4 |b|^2 (\lambda T)^2. \end{aligned} \tag{44}$$

Diagram (v) One particle has two compatible collapses dominating the collapse on the other particle.

$$\begin{aligned} P_{2v} &= |a|^2 \int_0^T \lambda |b|^2 dt \exp\left(-\lambda T\right) \exp\left(-\lambda(T+t)\right) \int_0^{T+t} \lambda dt' \exp\left(-\lambda t'\right) \\ &= \frac{3}{2} |a|^2 |b|^2 (\lambda T)^2. \end{aligned} \tag{45}$$

Diagram (vi) As in the last diagram, but the order of the first two collapses is reversed.

$$\begin{aligned} P_{2vi} &= |b|^2 \int_0^T \lambda |a|^2 dt \exp\left(-\lambda T\right) \exp\left(-\lambda(T+t)\right) \int_0^{T-t} \lambda dt' \exp\left(-\lambda t'\right) \\ &= \frac{1}{2} |a|^2 |b|^2 (\lambda T)^2. \end{aligned} \tag{46}$$

Diagram (vii) Similar to diagram (iii) with the same difference as between diagrams (ii) and (iv).

$$\begin{aligned} P_{2vii} &= |b|^2 \int_0^T \lambda |a|^2 dt \exp\left(-\lambda T\right) \exp\left(-\lambda(T+t)\right) \int_0^t \lambda |a|^2 dt' \exp\left(-\lambda(T-t+t')\right) \\ &= \frac{1}{2} |a|^4 |b|^2 (\lambda T)^2. \end{aligned} \tag{47}$$

Adding these probabilities together, gives

$$P_2 = |a|^2 + \lambda T |a|^2 |b|^2 (|a|^2 - |b|^2) - \frac{1}{2} (\lambda T)^2 |a|^2 |b|^2 (|a|^2 - |b|^2)(5 - 4|a|^2|b|^2), \tag{48}$$

which is the same probability as that obtained for the one-particle, two-peak wavefunction.

6 Magnitudes

The crucial parameter in the above discussion is

$$\lambda T = \frac{L}{c} \frac{N}{\tau_{col}}, \tag{49}$$

where τ_{col} is the collapse time for a single particle and N is the number of particles involved in the measurement apparatus. If we take $L = 10\,\mathrm{m}$ corresponding to roughly the largest separation in the Aspect experiments [10], and use the GRW value ($10^{16}\,\mathrm{s}$) for τ_{col} this becomes

$$\lambda T = \frac{N}{3} \times 10^{-23}. \tag{50}$$

It is clear that with a macroscopic apparatus this number could well be of the order of unity or larger. Hence, the possibility of detecting violations of the quantum probability rule certainly exist. On the other hand, the uncertainty in the value of the parameter τ_{col}, and the possibility of variation in the precise predictions of particular versions of the collapse models, e.g. as discussed in [6, 7, 8] etc., mean that it is not possible to rule out our local form of collapse models. In order to do this, or to see the new effects they predict, it would be necessary to do experiments in which L and N are as large as possible, and in which the ratio of $|a|$ to $|b|$ in the measured state lies in the middle of the range [0,1]. It would of course also be necessary to do a careful analysis of the actual measuring apparatus, rather than just modelling it by a 'pointer' as in §5 above.

It should be noted that the only necessary constraint on the collapse time of the apparatus ($\frac{\tau_{col}}{N}$) is that it is less than the time of perception (τ_{per}), which is certainly not less than $10^{-4}\,\mathrm{s}$. Hence we know $\lambda T > \frac{T}{\tau_{per}} \sim 10^{-3}$. There is ample space here for values of λT considerably less than unity, for which deviations from the quantum probability law would not have been seen. On the other hand, by using larger apparatus and/or larger values of L, the effects should certainly become observable.

7 Summary

An important contribution of the original GRW model was that it showed the *possibility* of defining a precise model in which collapse happens as a physical process in such a way that the tested predictions of quantum theory still held (This is independent of the issue of whether nature actually *chooses* this particular solution of the measurement problem).

In the same spirit we have shown here the possibility of defining a precise collapse model which is *local* and *Lorentz-invariant*. The issue of whether it is still consistent with all experiments is somewhat less clear, but as stated above the freedom in the

choice of parameters almost certainly means that it can be made consistent. To this extent we have shown that the widespread belief that Bell's theorem combined with the results of the Aspect et al. experiments [10] mean that any realistic model of quantum theory (apart from 'many-worlds' versions) must be explicitly non-local, is false.

References

[1] G.Ghirardi, A.Rimini and T.Weber, Phys. Rev., **D34**, 470 (1986)

[2] G.Ghirardi, P.Pearle and A.Rimini, Phys. Rev., **A42**, 78 (1990)

[3] P.Pearle, in *Sixty-Two Years of Uncertainty*, ed. A.Miller (Plenum, New York, 1990)

[4] P.Pearle, in *Quantum Chaos and Quantum Measurement*, eds. P.Citanović, I.Percival and A.Wirzba (Kluwes, Dordrecht, 1991)

[5] G.Ghirardi, R.Grassi and P.Pearle, Found. Phys. **20**, 1291 (1990)

[6] P.Pearle and E.J.Squires, Phys. Rev. Lett., **73**, 1 (1994)

[7] C.Dove and E.J.Squires, Found. Phys. **25**, 1267 (1995)

[8] B.Collett, P.Pearle, F.Avignone and S.Nussinov, Found. Phys. **25**, 1399 (1995)

[9] C.Dove, PhD Thesis

[10] A.Aspect, P.Grangier and G.Roger, Phys. Rev. Lett., **47**, 460 (1981) and Phys. Rev. Lett., **49**, 91 (1982); A. Aspect, J.Dalibard and G.Roger, Phys. Rev. Lett., **49**, 1804 (1982)